**MAT 243 Week 1/7 Written Homework  
1. (1 point) Is ”Johann Sebastian Bach is the greatest of all the Baroque  
composers” a proposition? Explain.**

No, this is an opinion. If there is no definite true/false answer, it is not a proposition.

**2. (1 point) Write the negation of ”Hikaru is taller than Yutaka”. Your  
(verbally given) negation must not contain any words or phrases that  
explicitly express negation, such as ”not”, ”it is untrue that”, ”is false”,  
”is incorrect”, etc.**

Hikaru is shorter than Yutaka

**3. (a) (1 point) Write the fully simplified negation of 3 < x ≤4.**

**(b) (1 point) Negate verbally: ”all people weigh at least 100 pounds.”**

Some people weigh less than 100 pounds.

**4. (1 point) Is the conditional statement ”If a human being has 7 heads, then  
they have 11 arms” true or false? Explain.**

This statement is true because the premise of the statement is false. Conditionals with a false premise are true.

**5. (1 point) Rephrase in contrapositive form: ”If you are taller than 6 ft, then  
it is unpleasant for you to travel in economy class.” Your contrapositive  
must not contain explicit references to negation. Assume that the negation  
of ”unpleasant” is ”pleasant”.**

If it is pleasant to for you to travel in economy class, then you are 6 feet tall or shorter.

**6. (5 points) Rephrase verbally in equivalent only if, sufficient, necessary,  
contrapositive and unless form: ”if we had an FTL drive, then we could  
visit the stars”.**

**Only if: If we could not visit the stars, then we did not have an FTL drive.**

**Sufficient: If we had an FTL drive, then we could visit the stars.**

**Necessary: If we do not have an FTL drive, we can not visit the stars.**

**Contrapositive: If we can not visit the stars, we do not have an FTL drive.**

**Unless: We could visit the stars unless we did not have an FTL drive.**

**7. (1 point) Write the formal negation of ∀x∃y(x > y). Your negation must  
not contain any explicit negation symbols.**

**8. (4 points) Use logical equivalences to simplify (p →q) → (¬p →¬q) until  
you have at most one occurrence of each variable p, q remaining. Identify  
all logical equivalences by name. You will not receive credit for a truth  
table solution.**

**(p →q) → (¬p →¬q) Given**

**By Equivalency By Equivalency By De Morgan By Distribution By Absorption By Equivalency**

**9. We have two mass storage devices, A and B with the same specs. Each can  
store N bits, but read/write operations are slow. We mitigate this problem  
by turning the two devices into a storage array: data is alternatingly  
written to A and B. The array can now store 2N bits, and read and write  
at almost twice the speed of the individual devices, assuming that the  
time it takes to send data to A and B is negligible, compared to the time  
it takes each device to write the data.  
There is however a flaw in this plan. Since data is distributed over two  
devices, the total chance of failure has now doubled. The failure of just  
one device causes the failure of the array.  
We could eliminate this problem by copying all of A’s data redundantly  
to an additional devices C, and all of B’s data to an additional device  
D. This insures against single and double drive failure, but at the cost of  
doubling the required amount of storage devices.  
We decide that simultaneous drive failure is sufficiently unlikely to ignore  
that possibility, and we will guard against data loss from single drive  
failure only. This requires only one extra device C, as follows:**

**We abstract each device as a list of bits, A = a1, a2, ..., aN , B = b1, b2, ..., bN  
and C = c1, c2, ..., cN .  
Then for each k = 1...N , C stores  
ck = ak XOR bk.  
Here, XOR is the bitwise exclusive or operation.**

**(a) (2 points) With this scheme, if bit k fails on device A or B (not both),  
it can be reconstructed from bit k on device C. Explain.**

**Consider all cases of Ak, Bk:**

|  |  |  |
| --- | --- | --- |
| **Ak** | Bk | **Ck** |
| **0** | **0** | **0** |
| **0** | **1** | **1** |
| **1** | **0** | **1** |
| **1** | **1** | **0** |

**In the case drive A failed, Anew = Bk XOR Ck**

|  |  |  |
| --- | --- | --- |
| **Bk** | **Ck** | **A­new** |
| **0** | **0** | **0** |
| **1** | **1** | **0** |
| **0** | **1** | **1** |
| **1** | **0** | **1** |

**Note that Anew is the same as the original Ak.**

**Additionally, this also works to recover driver B: Bnew = A­k XOR Ck**

**(b) (2 points) Would the same reconstruction property still hold if you  
used logical AND instead of XOR? Explain your answer.**

**No, using AND would require both inputs to be high for Ck to be used to recover the information.**

|  |  |  |
| --- | --- | --- |
| **Bk** | **Ck** | **A­new** |
| **0** | **0** | **0** |
| **1** | **0** | **0** |
| **0** | **0** | **0** |
| **1** | **1** | **1** |

**­Note, Ck ­values were changed to match the original values of Ak. Anew ≠ Ak following the rule, Anew = Bk AND Ck.**

**(c) (2 points) Would the same reconstruction property still hold if you  
used logical inclusive OR instead of XOR? Explain your answer.**

**No, using OR would allow for high value to be saved to Ck more often. Using the same example as in B following the rules Ck = Ak OR Bk, and Anew = Ck ­OR Bk**

|  |  |  |
| --- | --- | --- |
| **Bk** | **Ck** | **A­new** |
| **0** | **0** | **0** |
| **1** | **1** | **1** |
| **0** | **1** | **1** |
| **1** | **1** | **1** |

**Note that there are more high values saved in Anew.**

**10. (2 points) Is the statement ∃x∀y(xy = 0) true or false? The domain of  
discourse is the set of real numbers. Explain.**

**True, there exists x = 0 which makes any value of y in the domain of discourse equal 0.**

**11. (2 points) If P and Q are predicates over some domain, and if it is true  
that ∀x(P (x) ∨Q(x)), must ∀xP (x) ∨∀xQ(x) also be true? Explain.**

**No, say P(x) = x > 5 and Q(x) = x ≤ 5. If P(4) Q(6), both predicates are false. Whereas, ∀x(P (x) ∨Q(x)), would be true.**

**12. (2 points) Suppose P is the predicate defined by P (x, y) = x is friends  
with y, where x and y are people. (No one is considered to be friends with  
themselves.) Translate the formal expression ∀x∃y∃z(y 6= z ∧P (x, y) ∧  
P (x, z)) into English.**

**For every x, there is a unique y and unique z such that x is friends with y and z.**

**13. (2 points) Let P be defined as in the previous problem. Is ∀x∃y∃z(y 6=  
z →P (x, y) ∧P (x, z)) true or false? Explain.**

**False, it could be possible that x does not have any friends, or has one friend.**

**14. (4 points extra credit) The goal of this problem is to disprove a logical  
equivalence programmatically. Write a Python program that prints a com-  
plete truth table for (a → b) → (c → d) and (a → (b → c)) → d, and  
marks the rows where the two statements have different truth values. You  
have to build the truth table from scratch. Don’t use a build-in truth  
table generator. Show your program and the output.**